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Arguing Against Fundamentality

Abstract: This paper aims to open up discussion on the relationship between fundamentality and naturalism, and in particular on the question of whether fundamentality may be denied on naturalistic grounds. A historico-inductive argument for an anti-fundamentalist conclusion, prominent within contemporary metaphysical literature, is examined; finding it wanting, an alternative 'internal' strategy is proposed. By means of an example from the history of modern physics - namely S-matrix theory - it is demonstrated that (1) this strategy can generate similar (though not identical) anti-fundamentalist conclusions on more defensible naturalistic grounds, and (2) that fundamentality questions can be empirical questions. Some implications and limitations of the proposed approach are discussed.

Part 1. The Philosophical Background

Let me begin by highlighting two salient assumptions of much contemporary metaphysics.[1] The first concerns "an intuition commonly held by metaphysicians", namely the intuition "that there must be a fundamental layer of reality; that chains of ontological dependence must terminate; that there cannot be turtles all the way down."[2] Since ontological dependence relations ('ODRs') are standardly assumed to form partial

[1] It can of course also be found in canonical philosophy of science: see for example Oppenheimer and Putnam [1958].
[2] Cameron [2008], p1.

orderings, the assumption is that ODRs constitute *well-founded* partial orderings.[3] Thus, to take an example, if one takes *supervenience* relations with properties as their relata to constitute ODRs, the intuition is that there must exist a set of properties on which all other properties supervene, but that are not themselves supervenient upon anything.[4] Or if it is *mereological* relations with objects as their relata that constitute the ODRs in question, the intuition is that there must exist a set of objects that are sufficient to compose everything but that do not themselves have any proper parts. Underpinning this intuition is the "anti-gunk worry" that in a 'gunky' world in which every object has proper parts,

> composition could never have got off the ground. If the existence of each complex object depends for its existence on the existence of the complex objects at the level below, and if we never reach a bottom level, then it is hard to see why there are any complex objects at all… In Schaffer's charming phrase, 'Being would be infinitely deferred, never achieved'.[5]

The 'worry' is presumably analogous for any other ODRs one might identify.

The second assumption is that, whether conceived of as populated by objects or by properties, *the fundamental basis is physical in nature*. The most familiar contemporary proponent of this thought is probably David Lewis, who sees it as "a task of physics to provide an inventory of all the fundamental properties and relations that occur" in an assumed fundamental supervenience basis for this world.[6] The belief that it is physics' job to fill in the details of this basis is a pervasive one: Kim for example observes that "the bottom level is usually thought to consist of elementary particles, or whatever our

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[3] For example Schaffer ([2003], p500) "assume[s] that the priority relations among actual concrete objects form a well-founded partial ordering… Well-foundedness is imposed by requiring that all priority claims terminate."
[4] That supervenience relations *do* qualify as dependence relations is a matter of dispute. I will touch on this again below.
[5] Cameron [2007], p.6. The word 'gunk' as a term for objects all of whose parts themselves have proper parts was introduced by David Lewis in his [1991].
[6] Lewis [1999], p292.

best physics is going to tell us are the basic bits of matter out of which all material things are composed."[7]

The philosophical position outlined here – and which I take to be deeply entrenched – is therefore constituted by two elements: (1) an *a priori intuition* that the structure of dependence relations must be well-founded - that is, that a fundamental basis must exist; and (2) a *delegation to physics* in settling what that fundamental basis is like. But it will be immediately obvious that this state of affairs is deeply dissatisfying from a naturalistic point of view. Under the assumption that the fundamental basis exists, physics gets a role in saying what it is like; *that* the basis exists in the first place, however, is relegated to armchair contemplation. But surely we want physics to contribute to *every* aspect of our metaphysics if at all possible, not for it to be assigned piecemeal roles; we certainly do not want fundamental questions about the structure of reality to be answerable only to our intuitions about what must be so. The question with which we shall be concerned here is therefore that of whether physics can contribute not just to questions of the content of an assumed fundamental basis, but to the structure of dependence itself, and in particular with the question of whether physics can enable us to argue *against* the assumed well-foundedness of dependence relations. Can physics deny fundamentality?

Part 2 The Inductive Argument Against Fundamentality

Jonathan Schaffer has recently issued a challenge to the assumption that there exists a set of fundamental entities that ultimately compose everything. By reflecting on over a century of developments in the study of the structure of matter he argues that the more scientifically informed position does *not* after all sanction a belief in a fundamental basis at all – a conclusion that flies in the face of the standard assumption.[8] Though annotated

[7] Kim [1998], p15.

[8] It should be noted that Schaffer is *not* deploying his argument in favour of the idea that there is no fundamental level at all – cf. footnote 3 above. Given that his work since that publication has been primarily to argue for his 'priority monism', we can take the intention behind his argument to be to 'butter up' philosophers to the notion of grounding *from above.* Since I take it that fundamentality questions ought not to be settled *a priori* but

with facts and observations of various sorts, Schaffer's argument is easy to summarize: it is (what we might call) a meta-induction, and in a nutshell it is this.

> The history of science is a history of seeking ever-deeper structure. We have gone from 'the elements' to 'the atoms' to the subatomic electrons, protons and neutrons, to the zoo of 'elementary particles', to thinking that the hadrons are built out of quarks […] Should one not expect the future to be like the past?[9]

In other words, the fact that progress in the study of matter has largely consisted of instances of fractioning entities thought to be fundamental into the more fundamental entities they are dependent upon is being claimed to furnish an argument that it is better in keeping with the history of physics to deny the existence of a fundamental level. The claim seems to be that we have good inductive and naturalistic grounds for denying, in particular, *mereological* fundamentality, though this presumably has implications for fundamentality theses that are cashed out in supervenience-based or nomological terms.[10]

It should be immediately clear that if Schaffer's argument succeeds, it will be a remarkable result. The question of the infinite divisibility of matter was after all one of Kant's antinomies. A clear demonstration that one should not believe in fundamental entities would dismantle an edifice of prevalent contemporary (and ancient) metaphysical thinking in strikingly succinct terms. And there are some who believe that

should be a matter, if possible, for empirical enquiry, *I* certainly would not take any perceived lack of a basis 'beneath' to be evidence for one 'above'.

[9] Schaffer [2003], p.503. Note that the examples he chooses are not all on a par: that the relationship between 'the elements' and 'the atoms' is compositional in anything like the sense that atoms are composed of nuclei and electrons is far from clear. But we will not pursue this here.

[10] Callender [2001] notes his misgivings about the fact that Schaffer's discussion concerns particles and compositional relations. This, Callender feels, is illegitimate on the grounds that the 'fundamental particles' are today conceived of as *fields,* and that although fields are "in some sense infinitely divisible", they are only "horizontally" so. But it seems to me that this objection has, as it stands, yet to be fully made out: after all, particle physics *does* apparently recognise a distinction between fundamental and composite *fields* (insofar as, for example, there is currently an open question in "beyond the Standard model" physics of whether the Higgs field ought to be regarded as fundamental or as a composite of top quarks). But note too that Callender himself recommends that we be charitable and understand Schaffer's argument 'loosely' in less contentious, supervenience-based terms, and the argument I will adduce against Schaffer applies equally to such a construal.

it *does* succeed. Though citing discomfiture with certain of Schaffer's assumptions, Ladyman and Ross are tentatively supportive of the spirit of Schaffer's proposal, citing that

> Arguably we do have inductive grounds for denying that there is a fundamental level since every time one has been posited, it has turned out not to be fundamental after all.[11]

But I would not recommend that we join this club. However appealing though it may be, and though it should certainly give the fundamentalist pause, Schaffer's argument nevertheless fails to exemplify the naturalistic approach we seek. To cite a first problem (and one that Callender has highlighted), it is surely stretching the inductive evidence – a handful of cases – beyond breaking point to take it to support the in principle infinite amount of work that the argument needs it to do.[12] One may also get the feeling that there is something suspiciously question-begging about Schaffer's use of the evidence.[13] But while these both represent grave problems for Schaffer's whole approach, there is in fact a more pertinent and structural difficulty with it from a naturalistic point of view. Consider again the picture that Schaffer is offering us. It is a picture in which that which was thought to be fundamental is revealed as being in truth dependent upon other things. Again and again the fundamental basis changes, but what remains the same throughout is the *nature* of the dependence relation connecting the various levels, and

[11] Ladyman and Ross [2007], p178.
[12] Callender [2001] p3.
[13] This is not just Schaffer's problem: any historico-inductive argument aimed at establishing a fundamentality conclusion must be guilty of the same problem. To see this, note that Callender points out that given "the simple fact that science has (virtually) always gone for a fundamental level… [the] history of science does not support an infinite descent more than fundamentalism – if anything quite the opposite."[13] So which conclusion *does* the evidence support? Assuming that Callender is right that science 'virtually always' *does* posit a fundamental level, then one could object that the historical process that Schaffer describes of positing successive layers of more fundamental entities could be used to argue against the existence of a fundamental level *only if* we assume that the process of *refuting* that any such layer is fundamental will repeat forever – or, in other words, by assuming that there is no fundamental level. Likewise, given that the process Schaffer describes of successive refutations of fundamentality claims has taken place historically is as incontrovertible as Callender's claim that a sequence of fundamental levels has in many cases been postulated, one can argue that the historical evidence supports fundamentality *only if* one holds that the process of refutation must come to an end – or, in other words, that there *is* a fundamental level. It therefore appears that the argument for *either* conclusion on the basis of the historical evidence begs the question. And it is hard to see how any such meta-induction for or against fundamentality could avoid doing so.

indeed its *structure*. (One obviously cannot infer from an observed finite segment of a chain of partially-ordered dependence relations to its being infinitely long without already *assuming* that the relations will continue to form a chain in the hitherto unobserved regimes.) But since the fundamental is standardly *defined* as that which is ontologically dependent on nothing, it is presumably ontological dependence that constitutes the central concept in any fundamentality debate; if we want that debate to be *naturalized,* then we surely cannot permit questions of either the nature or the structure of that relation to be insulated from the jurisdictions of physics. It would surely, in any case, be *unwise* to make assumptions as to the dependence structure that will be suggested by the metaphysics of future physics: indeed, one could claim that the fact that the supervenience structure exhibited in composite quantum systems can be argued to be (in some sense) oppositely directed from that which would have been expected classically already provides us with confirmation that *a priori* assumptions regarding the dependence structure in hitherto unknown regimes are apt to go awry.[14]

In summary, then, the insuperable problem with historic-inductive arguments against fundamentality such as Schaffer's is that they necessarily rest upon speculative assumptions regarding the metaphysics of future physics that are surely at odds with the naturalistic agenda. It is for this reason that I propose that arguments against the existence of a fundamental basis *should always be formulated within the perspective of a physical theory*. These I call *internal* arguments against fundamentality.[15] Now, we must be clear at once that this is not some empty *a priori* prescription, for there genuinely have been theories whose internal logic can be used in support of anti-fundamentalist interpretations: it is *not the case* that theories of matter must assume fundamentality and it is only ever history that proves them wrong. To take a notable

[14] A classic paper on this is Teller [1986]. Note that it is controversial as to whether supervenience constitutes a genuine dependence relation; I myself in fact do not believe this, largely on the basis of Kim's criticisms (Kim [1993]). However, it is part of Kim's argument that supervenience and dependence are nevertheless "not entirely independent, for it seems that the following is true: for there to be property dependence there must be property covariation." (ibid. p148). If this is correct, it follows that the failure of supervenience in this case may be taken to indicate the absence of a dependence relation – and one that would have been expected classically.

[15] Such an approach in a sense simply parallels the principal argument cited for *committing* to fundamental entities, namely, that our best current physics supports it. According to the 'internalist' approach we should likewise *deny* commitment to fundamental entities when and only when our best theories recommend it. In the absence of clairvoyance, and given the difficulties expanded upon in note (13) above, I believe this is all we can reasonably hope to do.

contemporary example, the 'effective' approach to quantum field theory has been argued to support the idea that a commitment to a certain theoretical framework – quantum field theory without the renormalization postulate – can naturally lead one to deny the existence of a fundamental level.[16] There is also a growing body of literature on how dualities in string theory can undermine certain fundamentalist assumptions.[17] Here, however, I will focus on another example, and this will be the *S-matrix theory* of strong interactions.[18] Now - as I will be the first to point out - this may initially be deemed to constitute a rather odd staring-point from which to address the issue at hand: after all, both the effective approach and string theory represent live theoretical possibilities, whereas the S-matrix theory has barely seen the light of day since the 1960s. But my objective here is not to argue for the lack of a fundamental basis *as a feature of actuality,* but rather to study the features that internal arguments against fundamentality can or must have. Viewed in this light, the choice is very fitting since S-matrix theory is both well-understood and contains a number of theoretical and empirical features that are highly relevant for this purpose– indeed, the very means through which it ultimately came to be *rejected* will prove to be important to our project.[19] In particular, the S-matrix theory provides us with a demonstration of how physical theory can be marshalled in support of the idea that there is no *mereological* fundamentality (that our world is a 'gunk' world, if you will); since it is compositional structure that is taken to be the "central connotation" of dependence structure, this is a nice place to start.[20] We therefore begin the discussion of how anti-fundamentalist interpretations can be naturalistically wrought with this theory, and to this end we will proceed as follows. We will acquaint ourselves with the S-matrix framework for hadronic interactions and from there establish that the compositional relations amongst

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[16] See Cao and Schweber [1993] for a discussion of the concepts involved in an effective field theory, and the outlines of an argument as to how this new approach may support anti-fundamentalist conclusions. I hope to discuss my own interpretation of Cao and Schweber's argument, and a partial defence of it, in a companion paper.

[17] See for example Rickles [2011], section 3.2; Castellani [2009].

[18] This is also sometimes known as the 'bootstrap theory' of strong interactions.

[19] Of course, the study of S-matrix theory may be valuable for other reasons, particularly given that it was this programme that gave rise to string theory.

[20] Schaffer [2003], p500. I stress that with the term 'mereological', I do not for a moment wish to connote a commitment to any *a priori* theory of composition, such as a philosophical theory with 'fusion' and an *a priori* prescription on logical form - quite the opposite in fact, for below I will attempt to extract the appropriate logic of part-whole relations from the S-matrix's own assumptions. Furthermore, we should note that since the S-matrix theory concerns only the strongly interacting particles, it has nothing to say on the existence of fundamental particles of any other sort (such as leptons); however, the existence of even a proper subset of objects for which the 'chains of dependence' do not terminate is sufficient to refute the idea that the world possesses a mereologically fundamental basis.

hadrons may be understood to form partial orders. Three arguments against the well-foundedness of these orders will then be discussed, from which some general morals regarding internal arguments against fundamentality will be drawn. So with that in mind, let us introduce the S-matrix theory with a little of the background into which it was born.

Part 3 Introducing the S-Matrix

The S-matrix theory of the strong interactions was introduced into particle physics in the 1950s, largely in response to the fact that quantum field theory was at that time experiencing well-known and seemingly critical complications.[21] Even aside from the consistency problems that seemed to plague local field theories in general, the prospects for a workable field theory of the *strong* interactions seemed particularly hopeless due to the resistance to perturbative techniques that its large coupling was taken to imply. Fortunately, however, another approach was waiting in the wings to circumvent this impasse, one based directly on the scattering matrix, or *S-matrix*, that had been introduced into physics by Wheeler in 1937 (to be subsequently developed by Heisenberg).

The scattering matrix is, of course, the principal means by which particle physicists forge contact between their theories and phenomena. The elements S_{ij} of this matrix encode the probability of obtaining a state j of free particles as the output of a collision event given that state i – another state of free particles – serves as the input. Given that all that quantum mechanics predicts is these probabilities of measurement, the totality of such elements may be said to constitute the entire empirical output of the theory.[22] Thus one could argue that if one could find a method of reliably and (at least 'in principle') exhaustively computing these elements, one could lay claim to having an (in some sense) 'complete' quantum theory of the strong interactions. This in any case was

[21] Cushing [1990] is the canonical source for extended discussion of the background to the S-matrix theory and its subsequent developments. A more condensed discussion of the mathematical and dynamical essentials of the theory, as well as its origin and ultimate demise, may be found in Redhead [2005].
[22] Stapp notes that "general quantum theory makes no predictions beyond those made by S-matrix theory" ([1971], p1303).

the view of Geoffrey Chew, the chief architect of the theory that was to be developed. Chew took it that

> Since elements of the S-matrix describe all hadron experiments, ability to predict this matrix would constitute a complete hadronic theory.[23]

It was this conviction that led Chew to construct a whole approach to the strong interactions based on this object, and S-matrix theory was born.

Despite its predication on this seemingly purely empirical object, Chew's S-matrix theory differed from purely instrumentalist approaches because it attempted to supply both a realistic portrait of particle structure and a dynamical description by means of this matrix. The basic idea behind S-matrix theory is that certain constraints on the scattering matrix suffice to determine it uniquely, and that this method of determination constitutes a complete quantum theory.[24] The constraints placed on the S-matrix during its early phase were the following:

1. *Strong interaction forces are short range.*
2. *Superposition.*
3. *Unitarity.*
4. *Lorentz invariance.*
5. *Maximal analyticity 'of the first kind' – i.e., in the* linear *momentum variables.*

These five principles may be said to constitute the axioms of the early phase of S-matrix theory.[25] It is easy to say that axioms 1-4 may be said to have straightforward physical underpinnings. The short-range postulate is empirically evidenced and means that we

[23] Chew, [1968] p763.
[24] String theory, which issued from S-matrix theory, arguably retains this 'principle theory' approach.
[25] In expounding this theory I rely mainly on expositions from Chew and the textbook of Collins and Squires. There may therefore be concern that the presentation provided here is somewhat biased. However, since the philosophical conclusions I wish to draw from this theory primarily issue from the unitarity postulate and the analyticity postulates (one more is to be introduced below), and these are keystones of the theory as a whole and not tied to any particular presentation of it, I do not think that these choices materially affect my conclusions. However, an alternative and comprehensive presentation of the technical issues relevant to this paper may be found in Eden, Landshoof, Olive and Polkinghorne (1966).

can treat the states related by the S-matrix as essentially free.[26] The second and third introduce the essentials of quantum mechanics. The fourth is obviously essential insofar as the goal is to construct a relativistic theory of quantum mechanics (and note that here we have no choice but to construct a relativistic theory since the binding energies in strong interactions are comparable to the rest-mass energies of the constituent particles).[27]

The fifth, however, may seem rather out of place: it seems wholly *mathematical* rather than physical in character, so what it is doing as an axiom of a physical theory is far from obvious.[28] Chew himself was rather open about the fact that it was largely a postulate, admitting of no conclusive physical motivation. But however it is justified *ab initio,* the principle certainly earns its keep, for it turns out to be this that elevates the S-matrix – based approach to a genuine dynamical theory (in the eyes of its protagonists at least). To see why this is, note first that, in this context, 'analytic' means having only *isolated* singularities, and this has the purely mathematical property that it permits (via Cauchy's theorem) the expression of the amplitude in terms of the singularities in the various channels.[29] It can furthermore be shown that at a fixed centre of mass energy for the 'direct channel' (i.e. for the collision we are directly performing), the amplitude can be expressed in terms of the singularities corresponding to particles produced in the 'crossed' channels (i.e. those obtained from the direct channel by interchanging an output particle for an input anti-particle; see below). That is, with *s* standing for the square of the direct channel 4-momentum, *t* as one of the crossed channels and *u* as the other, and with *u* fixed at some value u_o, we have for the amplitude

$$A(s,t,u_0)=\frac{g^2{}_s}{m_s^2-s}+\frac{g^2{}_t}{m_t^2-t}+\frac{1}{\pi}\int_{s_b}^{\infty}\frac{\operatorname{Im}A(s',t,u_0)}{s'-s}ds'+\frac{1}{\pi}\int_{t_b}^{\infty}\frac{\operatorname{Im}A(s,t',u_0)}{t'-t}dt' \qquad \text{(Eq. 1)}$$

[26] Collins and Squires [1968], p7.
[27] Chew [1971a], p141.
[28] Though there is some connection with analyticity and 'the principle of causality', it is inapplicable here. There is a well-known *classical* connection between analyticity and the idea that scattered processes cannot happen before the particles are initially brought together. However, the problem with the derivation of this relationship is that it must assume both that the wave packets that represent quantum particles are localized *and* exploit precise values for their energy and momentum. It can therefore do little more than establish the need for causality in the classical limit. See for example Collins [1977], pp11-12. The analyticity postulate is further discussed in Redhead [1980], section 7.
[29] That is, the amplitude functions are assumed meromorphic; the basic point here is just that they contain none of the pathological behaviour associated with delta functions, step functions etc. (See Collins and Squires [1968], p68.)

These pole and branch cut singularities give contributions to the amplitude of the same form as those found (in the Born approximation) to be due to one-particle and superposed Yukawa potentials respectively.[30] Since Yukawa interactions are manifested by particle *exchange*, it can be therefore maintained that it is the exchange of the crossed-channel particles that supplies forces in the direct channel.[31] These are the forces required to generate the interactions that result in the production of new and qualitatively different states from the input channel. It is the seemingly purely mathematical property of analyticity, then, that delivers in the formalism the forces required to form new particles. It is therefore this postulate that may be said to supply the theory with *dynamics*.[32]

It can be easily demonstrated that the symmetry between the direct and crossed channel variables implies that the roles of the intermediate bound state and that of the force-carrying particle may be interchanged (insofar as particles of the same *type* may play either role). It is this feature that prompted Chew to also designate his theory as the 'bootstrap theory', since the particles that generate a given type of particle are in turn generated by *it*:[33]

> By considering all three channels on this basis we have a self-determining situation. One channel provides forces for the other two – which in turn generate the first.[34]

In order to make the story here a little more concrete, let us take a simple example and show what is going on diagrammatically (a method Chew often used in his

[30] See Chew [1962], p31.

[31] "The forces producing a certain reaction are due to the intermediate states that occur in the two 'crossed' reactions belonging to the same diagram. The range of a given part of the force is determined by the mass of the intermediate state producing it, and the strength of the force by the matrix elements connecting that state to the initial and final states of the crossed reaction." Chew [1962], p32.

[32] Note that this improvement on Heisenberg's original S-matrix theory is predicated on an explicit formal analogy with a result from the rival QFT (in terms of which both the Born approximation and the Yukawa interaction were originally formulated.) Whether S-matrix theory could have been a feasible approach to particle physics without a significant amount of borrowing from QFT seems highly doubtful, but it is not the success of S-matrix theorists in their ambition of providing a genuine *alternative* to QFT that occupies me here.

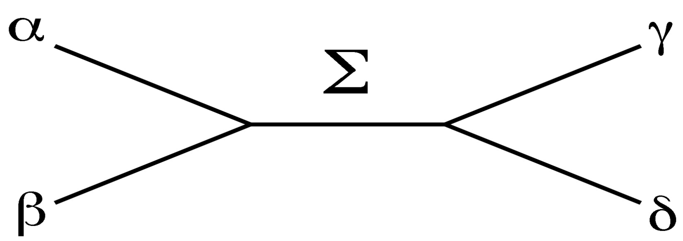
[33] In case of confusion, I emphasize that this applies only at the level of types, not tokens.

[34] Chew [1962], p32.

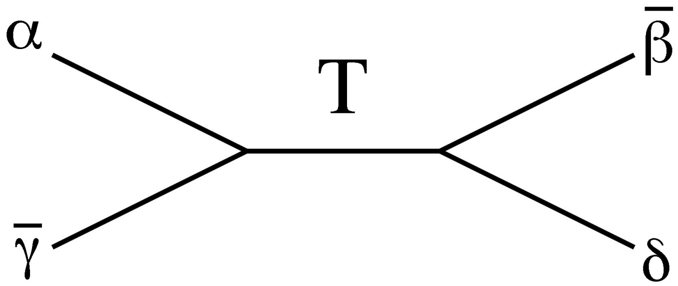
expositions).[35] Let us stick to a low-energy approximation in which only one one-particle intermediate state can be produced in each channel. Suppose the direct channel corresponds to the reaction

$$\alpha+\beta \rightarrow \gamma+\delta$$

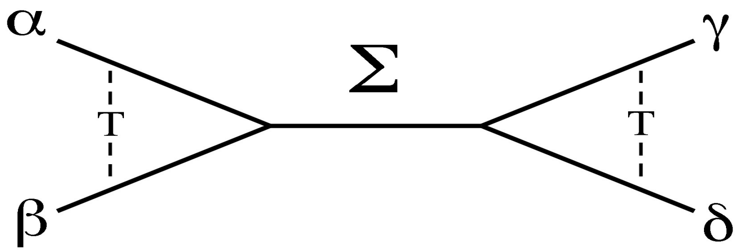
and that the single-particle intermediate state is particle Σ. Working in momentum space , we can represent this as



A crossed channel corresponding to this process then would be



It follows that the force required to bind Σ from the input particles is provided, in part, by the exchange of the particle T between by α and β, and γ and δ.



[35] For a detailed exposition of the use of diagrams in S-matrix theory see Kaiser [2005], chapter 9.

One-particle states produced in this way Chew refers to as 'bound states'.[36] All such particles are (incontrovertibly) regarded as *composite*. On the other hand, a particle that is *not* composite, and hence does *not* have its origin in an interaction, is in Chew's terminology an 'elementary particle'. Since we will construe fundamentality in compositional terms, these we may consider as the *fundamental* particles. It was a belief of the S-matrix theory's chief protagonists that that the theory implies that no such fundamental hadrons exist and we shall see presently some of this reasons behind this claim. But the immediate question that must be addressed at this point is what these composite particles may actually be said to be *composed* of. To the uninitiated in particle physics, a natural answer in the case of the intermediate state Σ above might be that it is simply the input particles α and β; this after all is what it is intuitively created from (though one must not forget the role of the binding particles.) But of course this 'obvious' choice is compromised by the fact that Σ does not decay *back into* α and β (that is, the reaction is *inelastic*). An examination of Chew's concept of a composite is clearly called for at this point; with that in hand we will be in a position to deduce the structure of compositional relations appropriate to S-matrix theory, and from there address the arguments as to their non-well-foundedness.

Part 4 The Concept of a Composite in S-Matrix Theory

Chew is unambiguous in his statement of what a composite particle is: a composite particle in S-matrix theory is "a bound state of those channels with which it communicates", where a channel is "any collection of more than one particle," and "'communicating' channels are nuclear states that possess all the same quantum numbers as a particular particle."[37]

[36] Of course *two* types of composite particles, bound states and resonances, are recognized by particle physics. The difference between the two is that resonances have a mass greater than or equal to the mass of the particles which go into the reaction from which it arises, and bound states have a mass less than the masses of the input particles. This entails an important *practical* difference between the two, since only resonances can be observed in scattering process such as the one sketched above (though note that the bound state contribution to the observed amplitude can nevertheless be detected: see Chew [1966], p99, footnote, for references). But since it is simply one of stability, in particle physics in general the distinction is not considered fundamental. In keeping with Chew's usage, we therefore subsume both types of composite particles under the banner of 'bound states'.

[37] A discussion of the S-matrix concept of composite particles from a metaphysical point of view, and with an eye on the parallels with Leibniz, may be found in Gale [1974].

Particles of any given type, then, are taken to consist of collections of particles such that they, in the aggregate, have the quantum numbers of the particle. Note that since the definition of a channel concerns only the sum of the quantum numbers of the particles involved, the property of being a certain channel is closed under the addition of particle – anti-particle pairs. Therefore particles of any one type can feature as constituents of a particle of any other type (provided of course that the latter is indeed composite).[38] In other words, bound states of any given hadron type T may contain constituents drawn from *any* hadron type whatsoever (including T itself).

Returning to our question, then, of what exactly Σ consists, it is clear that there is no unique answer - for any decomposition with the right quantum numbers will do. Now, one might initially feel dissatisfied by this statement: one might be tempted to think that if there are objectively existing composite particles, there must be a unique objective fact regarding what a given particle – say our particle Σ – consists of. But this would be mistaken, for quite generally, and as Lewis states, "a whole divides exhaustively into parts in many different ways".[39] For example, simple combinatorics tells us that a mundane object of experience, with (something of the order of) Avagadro's number of molecules, will admit an *enormous* number of possible decompositions and, as Lewis points out, the best that one might hope for in such a case is that, "if we distinguish some parts of a fusion as *nice* parts, then a fusion will have a unique decomposition into nice parts."[40] And as an example of what Lewis means by 'nice' parts, he cites the *mereological atoms* of the object concerned, so that one might hope that an ordinary object at least divides up into 'elementary particles' in some unique way. But the reason we are interested in S-matrix theory is precisely because it was taken to imply that *no* particle was mereologically atomic and hence this cannot be appealed to here. Nor – to quote more Lewisian terminology – can we distinguish various parts as 'more natural' than others. For given that all hadrons are composed of hadrons of every other type, all hadrons - from the pion to the uranium nucleus - are regarded as on the same

[38] This assumes that binding energies are limitless – an assumption reflected in the range of the integral in (Eq.1).
[39] Lewis [1991], p5.
[40] Ibid. p22.

ontological footing and hence all equally 'natural'. Hence there is no reason to be dissatisfied with the enormous variety of decompositions on offer: it is nothing unusual, and in the absence of fundamental or otherwise privileged parts it cannot in any case be avoided. It is, however, *crucial* to note that a composite particle is emphatically not simply a 'fusion' of its constituents (to quote a term by which composition is often described in analytic metaphysics). The constituents *must be interacting* with one another by means of particle exchange in order for a bound state to form, for otherwise we have only free particles. The bound state exists for only so long as these interactions take place.

We are now in a position to sketch a definition of 'parthood' in S-matrix theory and show that it partially orders the set of hadrons. From the definition of a hadron as "a bound state of those channels with which it communicates", we know there are two necessary conditions on being a constituent of a token composite particle: x is a *composite* (i.e. a *bound state*) of particles $y_1 \ldots y_n$, where $n \geq 2$, just if the following conditions are satisfied:

1. $\sum QN(y_i)=QN(x)$, where '$QN(x)$' denotes 'the quantum numbers of x', etc., and '$\sum$' the sum over i;
2. The $y_1 \ldots y_n$ are interacting to give a net strongly attractive force.

Let us write 'x is a composite of particles $y_1 \ldots y_n$' as '$x=B(y_1 \ldots y_n)$' where 'B' denotes 'in a bound state'. Since a composite object is a composite of *all* of its components, we should impose that the decomposition into the object's constituents is *maximal* (i.e *exhaustive*):

3. If $x=B(y_1 \ldots y_n)$, then $x \neq B(y_1 \ldots y_n, z)$ for any $z \neq y_i$, $y_i \in \{y_1 \ldots y_n\}$.

These three conditions are all necessary. But they do not yet seem to be sufficient, for the above conditions do not ensure that they are the parts of particle x of hadron type T and not of some $x' \neq x$ also of type T. But the ability to specify a further condition that delineated the analysis of composition to the level of distinct tokens of the same type clearly presupposes an analysis *of* the distinctness of two tokens of the same type. Since our primary purpose here is not with the issue of quantum individuation but to make contact with philosophical arguments against the existence of fundamental entities, here we will simply take the distinctness of tokens as primitive and leave the analysis of this distinctness to another occasion (if indeed there can be one that is appropriate to this theory). So to ensure that the compositional analysis applies to these (by assumption) distinct tokens, let us just put in by hand the *uniqueness of composition*:

4. If $x=B(y_1...y_n)$, then for no $x'\neq x$ is $x'=B\{z_1...z_n\}$ if $\{z_1...z_n\}=\{y_1...y_n\}$.

These four necessary conditions now seem to be jointly sufficient.

From this analysis of a composite, we may define what it is for y to be a constituent or *part* of a particle x, or yPx:

$$yPx \text{ if and only if } x=B(y_1...y_n) \text{ and } y\in\{y_1...y_n\}.$$

With this in place, it is now easy to show that the parthood relations appropriate to S-Matrix theory form partial orderings. The crucial ingredient is the observation that the existence of the composite implies the existence of its parts but not *vice versa*. For if $x=B(y_1...y_n)$, then (since they are identical) the existence of x implies the existence of $B(y_1...y_n)$, and by the 'adjective drop' inference, the existence of the $y_1...y_n$ *in a bound state* implies the existence of the $y_1...y_n$; hence every $y_i\in\{y_1...y_n\}$.[41] Therefore the existence of a composite particle implies the existence of all of its parts. The converse however is not true: the existence of the $y_1...y_n$ is not sufficient for x, since the particles in the set may be free, violating condition 2, and so no $y_i\in\{y_1...y_n\}$ is sufficient for x

[41] For a discussion of the limitations of applicability of the adjective drop inference, see Schaffer [2009], p356.

either. Let us call this asymmetry between parts and wholes 'the asymmetry of existence'.

Given the asymmetry of existence, it follows immediately that parthood relations are asymmetric. For assume otherwise: that is, assume that we have that $x=B(y_1...y_n)$, and that, for some $y_i \in \{y_1...y_n\}$, we also have $y_i=B(x,z)$, for some z. Then we have that (1) y_iPx and that (2) xPy_i, so that parthood is symmetric. So by (1) and the asymmetry of existence, x implies y_i and y_i does not imply x; and by (2) y_i implies x. But this is once again contradictory.

Irreflexivity follows similarly. For assume that $x=B(x,z)$, for some z so that xPx and parthood is reflexive. Then again via the 'adjective drop' inference we have that x (considered as the composite) implies x (considered as the part); but then given the asymmetry of existence x (the part) does not imply x (the composite). But this is contradictory.

To establish transitivity, what we must show is that, if yPx and zPy, then zPx. By the definition of 'is a part of', this is equivalent to if $x=B(y, y_1...y_n)$, for some $y_1...y_n$ ($n\geq1$), and $y=B(z, z_1...z_m)$ for some $z_1...z_m$ ($m\geq1$), then $x=B(z, z_1...z_m , y, y_1...y_n)$.

But the definitions of x and y mean that $x=B(B(z, z_1...z_m), y_1...y_n)$, and by utilizing the adjective drop inference once again we have $x=B(z, z_1...z_m, y_1...y_n)$. And so (2) is satisfied too.

What we have arrived at, then, is that the parthood relations appropriate to S-matrix theory *partially order* the constituents of hadrons. What we have therefore done is make contact with the sort of mereological ordering that Schaffer alludes to, though on wholly internal grounds; if we are to argue that these partial orders are non-well-founded, we must now show that the theory implies that there are no particles that do not themselves have parts.[42] That this was the case was the opinion of the theory's founder, Chew, for in his opinion hadrons are citizens in

> a democracy governed by Yukawa forces. Each strongly interacting particle is conjectured to be a bound state of those channels with which it communicates, owing its existence entirely to forces associated with the exchange of particles that communicate with 'crossed' channels. Each of these latter particles in turn owes *its* existence to a set of forces to which the original particle makes a contribution. In other words, each particle helps to generate other particles, which in turn generate it.[43]

Here we meet Chew's colourful neologism of 'particle democracy'. A translation might be in order:

> If one wishes to relate this idea of particle democracy to the older language of bound states or composite particles, it amounts to saying that each particle is a composite of all the others.[44]

But of course, nothing in our *description* of what it is to be a composite particle implies that all particles *are* in fact composite. What we must do now is turn our attention to the arguments as to *why* S-matrix principles were taken to imply precisely this, and hence why it is that the S-matrix theory provides us with an example of an internal argument against fundamentality.

Part 5 S-Matrix Arguments Against Fundamentality

[42] I would like to stress again that even although a partial order – that which is simply assumed in standard philosophical mereology - has been arrived at, we did not get to this point by *a priori* speculation. Rather, we obtained this result by following through the logic of *sui generis* principles of physical composition that are to be found in this theory of nuclear physics.

[43] Chew [1964a], p34 (though this quote is repeated *verbatim* in countless other places).

[44] Martin and Spearman [1970], p8.

A variety of arguments against fundamentality can be found circulating in the S-matrix literature and in what follows we shall look at three of them. In increasing order of sophistication, they are the *argument from superfluousness*, the *argument from self-consistency* and the *argument from analyticity*. It will help to take these in order, so we begin with the first.

5a The Argument from Superfluousness

The first major reason for the disavowal for elementary particles is that the structure of the theory does not *a priori* require them. Whilst this may sound like a very weak motivation, it is nevertheless the case that Chew's S-matrix is rather unusual in being a theory of particle dynamics which does not require an *a priori* specification of the properties of particles. Consider for example how one would approach hadron dynamics from the Lagrangian perspective – say that of pion-nucleon scattering. The only relativistic Lagrangian which leads to consistent results in this case is

$$L = L_0 + ig\overline{\psi}\gamma_5\psi + \lambda\phi_i^2\phi_j^2$$

where *Lo* gives the free Lagrangian.[45] (Here *g* is a coupling constant associated with the nucleon and λ that with the pion.) As elements of the fundamental Lagrangian, the nucleon field and the pion field each come as a package complete with their spins and masses; the handle on this equation will then be turned to arrive at the composite structures which the pion-nucleon reaction may give rise to. But since this Lagrangian is the basis of all deductions in the theory, these properties cannot *derive from* anywhere else in the theory. Hence Lagrangians by design require "arbitrarily assignable components in a theory", or, in Chew's words, "fundamentons."[46] The very act of adopting a Lagrangian-based approach therefore concedes that there are physical

[45] See Cushing [1990], p132.

[46] Chew [1971b], p2334; Likewise, Chew writes of electromagnetism: "Whether one speaks of the photon or of the electromagnetic fields, there exists an *a priori* central component of the theory whose existence is accepted as given – not explained as a necessary consequence of general principles." (Chew [1966], p96.)

facts of central importance which cannot be explained by the theory and hence may be said to be 'arbitrary' (from the theory's perspective at least). Any elementary particles that are fed directly into them then count amongst the 'fundamentons' – in contrast with the composites they can be used to derive.

Contrast this picture with that furnished by the S-matrix. It turns out that – just as in Lagrangian field theory – there *is* a methodologically central equation in this approach, namely the *unitarity* equation:

$$2\,\mathrm{Im}\,A_{ij} = \sum_n A_{in}^{+} A_{nj}. \qquad \text{(Eq. 2)}$$

Veneziano for example calls unitarity "the fundamental dynamical condition" of S-matrix theory.[47] But this is more of a *schema* than an individual equation, since any particle whatsoever – including those that are incontrovertibly non-fundamental – can feature in the states *i* and *j* in just the same way. So although this equation is in some ways the most analogous to the Lagrangian above in terms of the central deductive function it performs, does not privilege particles in the way the latter does. This is not to say that *nothing* in the theory qualifies as 'arbitrary', however, for the origins of the *axioms themselves* are left largely unexplained. Chew writes for example that

> The superposition principle is accepted on an *a priori* basis and not explained. In other words, we take for granted the existence of a quantum world.[48]

Thus certain things *are* treated as fundamental in this theory, but they are *principles*, not *particles*. And if the structure of the theory does not *require* fundamental particles in the way other theories might, then a simplicity principle might be brought to bear to argue against their inclusion: for is it not a general rule of thumb that whatever is *superfluous* to a theory should not be countenanced by it?

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[47] Veneziano [1969], p35.

[48] Chew [1971b], p2331. Likewise, for Chew "it is pointless to seek the origin" of why useful functions usually turn out to be analytic (Chew [1966], p2)

S-matrix theory therefore had good methodological grounds not to countenance the existence of fundamental particles. However, given that what is at stake is so central a supposition of so much modern scientific and philosophical thinking, one would ideally like rather stronger motivations for dissenting on fundamentality. Methodological considerations in general, after all, can only carry so much ontological weight, and indeed many would take the even-handedness outlined above to recommend at best agnosticism. Fortunately for S-matrix theorists, however, two further arguments were waiting in the wings that suggested that not only was the theory compatible with the absence of fundamental entities but that the dynamics positively prohibited them. The first of these we may call 'the argument from self-consistency'.

5b The Argument from Self-Consistency

It turns out that it is not only the non-privileging nature of the unitarity equation that may render it inhospitable to the fundamentalist. It is also its highly *holistic* implications that are taken to suggest democracy. To get a sense of this connection, we must recall Chew's assimilation of fundamentality with arbitrariness: he holds that "by definition, a fundamental component is one that is arbitrarily assignable."[49] What this amounts to in this context is that the *mass* of a particle and its *couplings* to other particles are arbitrarily assignable, since these properties are functions of the binding energy and hence the sort of thing that can in principle be deduced from the S-matrix dynamics.[50] Now it is clear that *any* theory that aspires to describe complex structures cannot allow the properties of *all* of the particles it describes to be 'arbitrarily assignable' in this sense (including the more conventional theories so disparaged by Chew). S-matrix theory is no different in this respect. Here is how one of the textbooks puts the matter:

[49] Chew [1970], p23.
[50] A major shortcoming of the theory was that the properties of isospin and hypercharge encoded in the SU(3) flavour algebra could not be explained, all speculative talk of 'bootstrapping symmetries' aside.

> It would be surprising if *all* the poles could be specified arbitrarily. For instance suppose we include the neutron and proton poles in the S-matrix. We would then expect the deuteron pole to be generated by the 'force' between these two particles, so there should be no need to put it in beforehand. Our expectation about this is clearly based on the feeling that the deuteron is a composite particle, and that composites should be consequences of the theory, not part of the postulates. In quantum-electrodynamics one has to specify the masses and charges of the electron and positron, but not those of positronium, which can be calculated. To add to the theory the requirement that the positronium mass take some particular value other than the experimental one would certainly be inconsistent. A theory of strong interactions which enables one to specify the masses and couplings of all the particles arbitrarily is almost certainly similarly contradictory.[51]

It is here that we meet the connection that S-matrix theorists forged between fundamentality and *consistency*. Where we have composite particles, their properties should be deducible from the theory; *stipulating* some value for them and putting it in by hand is very likely to result in inconsistent results. In the limiting case in which *all* particles are composite, as was Chew's belief, the inference seems to be that *no* parameters should be arbitrary and *all* should be derivable from the others in a self-consistent or 'bootstrapping' way. As a textbook put it,

> Intuitively, it seems clear that if all the hadrons are to be composites of each other, and all the forces are due to the exchange of particles, then some form of self consistency is necessary…[52]

But if we want an example of a theory that feasibly provides the requisite degree of self-consistency, then the S-matrix fits the bill if anything does: the constraints imposed on

[51] Collins and Squires [1968] p33, italics added.
[52] Collins [1977], p73.

the S-matrix, particularly the unitarity condition, place *enormous* self-consistency requirements on the theory. Let us see in more detail how this works.

As already mentioned, the purpose of S-matrix theory is to compute the scattering amplitude for all strong-interaction processes. The imaginary part of this amplitude (in the physically possible regions of the momentum variables) is given by the unitarity equation, which I will repeat here:

$$2\,\mathrm{Im}\,A_{ij} = \sum_{n} A_{in}^{+} A_{nj} \qquad \text{(Eq.2)}$$

It immediately follows from the structure of this equation that the amplitude governing any one process is a function of the amplitudes for transitions to any state to which the original state can be connected consistently with the various conservation laws. Thus, as the energy tends to infinity, any one transition becomes a function of all possible processes with the same quantum numbers. And since this property is closed under the addition of particle-anti-particle pairs, eventually a contribution from *every type of particle* will appear.

Now consider again the Mandelstam representation of the scattering amplitude, Eq. 1, that is obtained via Cauchy's theorem. The Mandelstam representation gives the amplitude for a reaction in which the *s* channel energy is fixed as a function of the singularities – poles and branch cuts – in the crossed channels (though here for simplicity we are holding one crossed channel energy constant). Consider first of all the pole singularities. Their positions are given by the masses of the direct- and cross-channel one-particle intermediate states and their residues are identified with their couplings to the input and output channels. The branch cut singularities, on the other hand, contribute to the amplitude a function of the discontinuity across the cuts, which (in physical regions) is equal to the imaginary part of the corresponding scattering amplitudes. But these in turn are given by the unitarity equations schematized above in Eq. 2, and hence (for the reasons given a moment ago) in terms of all those states that can be connected to these channels consistently with the conservation laws.

We therefore see that a calculation of the amplitude for any one reaction incorporates (1) the scattering amplitudes for all reactions with the same quantum numbers as the reaction in question, and (2) the scattering amplitudes for those reactions obtained from the reaction under consideration by crossing. And we know that as the energy gets high enough, eventually every type of particle can be expected to contribute to both. This means that even in the case of the simplest reaction, pion-pion elastic scattering,[53]

> The end result is that a full knowledge of the forces governing pion-pion scattering amplitude involves a knowledge not only of the pion-pion scattering amplitude, but also of practically every other strong interaction. Thus, what we have is not a self-consistency condition on the pion-pion amplitude by itself, but a set of consistency conditions inter-connecting all strong-interaction amplitudes.[54]

In other words, to understand one particle, one essentially has to understand all. As Chew put it,

> A 'bootstrapped' S-matrix contains an infinite number of poles and no single one can be completely understood without an understanding of all the others.[55]

The principal significance of this feature for our purposes is simply that, given the perceived 'inverse' relationship between self-consistency and the number of arbitrarily assignable parameters, the very high self-consistency requirements on S-matrix theory could be taken to suggest that there were no particles with 'arbitrarily assignable' properties and hence no fundamental particles. As Chew puts it,

[53] This reaction is particularly simple because it is 'closed under crossing', i.e. all channel obtained by crossing are identical.
[54] Omnès [1971], p300.
[55] Chew [1968], p765.

> In this circular and violently non-linear situation it is possible to imagine that no free parameters appear and that the only self-consistent set of particles is the one we find in nature.[56]

However, the high degree of holism has other consequences – the most obvious of them being that it will likely prove impossible to actually *solve* the nested system of unitarity equations implied by the S-matrix.[57] Since the argument from self-consistency is predicated on the precise form of this solution, pending an actual solution of these equations no further progress can be made on the speculation that the S-matrix precludes fundamental particles.

But things in fact seem to be worse than that: not only does it apparently transcend human capability to show that the 'self-consistency' hypothesis is true, it can be demonstrated that, under very general conditions and in the absence of any further postulates, it is *not* in general satisfied. The Mandelstam expression for the amplitude is obtained, it turns out, under the assumption that the amplitude disappears asymptotically. But this is not in general a realistic assumption; what is more often found is that the asymptotic behaviour is only power-bound.[58] This means that *subtractions* need to be performed in order to restore convergence to the Mandelstam amplitude. But this means that, where N is the power of the asymptotic divergence, we must add to the amplitude a polynomial of order $N-1$ and thus introduce $N-1$ undetermined subtraction constants into the amplitude. The presence of these undetermined constants means that the amplitude is *not* in general determined by the unitarity equations.[59] Thus despite the *intuition* that the highly holistic character of the S-matrix prohibits undetermined parameters, and hence may be taken to exclude

[56] Chew [1964a], p34.
[57] As one textbook puts it, "solutions of the unitarity equation involves solution of infinite sets of coupled, non-linear, singular, integral equations… one would have to solve the entire strong-interaction problem in one fell swoop."Collins and Squires [1968], p140. Of course, approximations could be and were made in deriving predictions, though their validity was often largely unknown.
[58] See Chew [1966], p60, for a discussion of the experimental evidence.
[59] "Requiring that the amplitude satisfy maximal analyticity of the first kind, with all the singularities given by the Landau-Cutkosky equations, is not necessarily sufficient to determine it completely. It would be sufficient if it were known to vanish suitably at infinity, but otherwise subtractions, which may introduce arbitrary parameters, are needed." (Collins and Squires [1968], p30.)

elementary particles, the asymptotic behaviour seems to introduce an ambiguity in the representation that suggests this may not be the case after all.

But the game is not up yet. It turns out that the ambiguity can be effaced with an *extension of the analyticity postulate*, an extension that was desirable on a number of independent grounds. Furthermore – and perhaps remarkably - it is this posit that enables S-matrix theory to translate the intuitions concerning the absence of fundamentality into precise empirical predictions. Though this is the most technical tranche of the argument, let us therefore turn to how this comes about. (A summary of the results will be presented in the last section.)

5c The Argument from Maximal Analyticity

It is with the extension of the analyticity postulate to cover not just linear but angular momentum that the S-matrix theory reached its apogee. In fact, the absence of angular momentum from the initial five postulates could be regarded as a conspicuous one, for the decomposition of probabilities into partial wave amplitudes – amplitudes for specific values of angular momenta – was already an essential part of the toolkit in non-relativistic scattering theory. Not only does this decomposition permit individual waves to be analyzed, it also permits partial diagonalization of the unitarity formulae and hence offers a great deal of simplification of the "baffling" unitarity equations. It was therefore "inevitable that the angular momentum decomposition should receive major attention" in the S-matrix theory.[60]

However, it was clear from the start that the (non-relativistic) partial wave series developed by Wigner and already familiar to particle physicists would not do, since it did not produce amplitudes which were compatible with crossing – a central plank in the S-matrix dynamical scheme.[61] It could be shown, however, that this could be resolved by extending angular momenta to complex values – something Tuillio Regge

[60] Chew [1966] p40.
[61] See Barone and Predazzi [2002] , p.87 for a modern presentation of these issues.

had studied in detail only a few years before.[62] What he had shown was that the partial wave amplitudes had singularities in the complex angular momentum plane at physical (integral or half-integral) values, and that such poles were 'moving poles' (i.e. functions of the energy). The angular momenta of bound states of systems which were solutions of the Schrodinger equation were thereby shown to be connected by smooth functions or 'Regge trajectories', denoted by $\alpha(E)$, and the incorporation of complex angular momenta into scattering theory has since been referred to (and is well known) as 'Regge theory'. Its applicability in the context of the strong interactions was essentially a conjecture, since there was no equation of motion whose analyticity properties could be explicitly studied. But this conjecture both helped in solving in an elegant way a number of theoretical difficulties and – most importantly – was also sustained by a rich edifice of phenomenological evidence. Furthermore, it was with its assimilation that the companion notions of arbitrariness, compositeness, and analyticity finally traced out an articulated set of relationships.

The new postulate that the amplitudes should admit continuation in angular momentum (ℓ) presupposes that the amplitudes should be analytic functions of ℓ. 'Maximal analyticity' is understood in the case of angular momentum perfectly analogously to the case of linear momentum: the amplitude should admit continuation to complex ℓ-values with only such singularities as are demanded by unitarity. This sixth postulate of *maximal analyticity of the second kind* was therefore a natural extension of the postulate already present, and its inclusion completed the architecture of the S-matrix.

The story that begs to be told here is rather long and detailed, but here we shall have to content ourselves with just the crucial steps in the reasoning surrounding the incorporation of the postulate and the subsequent empirical and metaphysical implications.[63] The postulate of maximal analyticity amounts to the hypothesis that the partial wave amplitudes can be analytically continued to complex ℓ for all physical (real and integral or half-integral) ℓ. Now, it can be demonstrated that an analytic function

[62] Regge [1959], [1960].
[63] More details on what follows can be referred to in Collins and Squires [1968] and references therein.

for the partial wave amplitudes exists for all ℓ greater than the power of the divergence of the Mandelstam representation (mentioned above). This function is given by the *Froissart-Gribov representation* of the amplitude. The validity of the postulate, which postulated continuation for *all* ℓ, was therefore tightly bound up with the power of this divergence. It was then proved that unitarity demanded that this power, in the region of the amplitude in which the centre of mass energy in a given channel lay below zero, could be no greater than one, and this result was then extended to cover the entire Mandelstam plane. This *Froissart bound* on the power of the divergence therefore established that the possibility of the invalidity of the analyticity postulate was highly constrained, since a unique analytic continuation of the amplitude to complex values of ℓ for all ℓ >1 was demonstrably possible.

The significance of this was twofold. First of all, it could be shown that the singularities of those partial waves that *did* admit of analytic continuation – the 'Regge poles' – occurred at physical (integral and half-integral) ℓ values; secondly, and, crucially, it could be shown that these Regge poles *had a Breit-Wigner form*. But this is the form, familiar from the earliest days of nuclear physics, which was known to correspond to bound states and scattering resonances – or, in other words, to composite particles. Appearing as a Regge pole in the appropriate partial wave amplitude was therefore put forward as an *operational definition* of a non-elementary particle in relativistic theory, and *the particles corresponding to Regge poles were in this way established as composite.*[64]

An immediate consequence of all this was that *there could be no elementary particle with spin greater than one.* Thus analyticity in ℓ placed stringent constraints on the properties any fundamental particle may have. Nevertheless, and this highly significant achievement in constraining fundamentality notwithstanding, it remained that the status of particles with spins of ℓ=0, ½ and 1 were left hanging by the existence of the Froissart bound. Since the constraints on fundamentality would have to be extended

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[64]"The [original] pole-particle correspondence fails to distinguish between 'elementary' and composite' particles, but… Fraustchi and I conjectured that Regge asymptotic behaviour might be used in the relativistic hadron S-matrix to *define* 'compositeness'." (Chew [1970], p.764, italics added. See also Gribov [2003], p54.)

into the region of these low values if 'nuclear democracy' was to be sustained, demonstrating that analytic continuation into the lowest partial waves was possible became a pivotal problem for the theory.[65] However, no general method for doing so was established.[66] That the amplitude was an analytic function for *all* ℓ therefore entered and remained in the theory as a postulate.[67] *It was this hypothesis regarding the singularity structure that became the mathematical correlate of the metaphysical hypothesis that no particle was fundamental.*

> Let us postulate that the Froissart-Gribov amplitude can be continued to all physical [ℓ] values… and that the actual physical amplitudes are thereby always achieved. This conjecture… we shall designate as maximal analyticity of the second degree. It is equivalent to the concept of nuclear democracy…[68]

A consequence of the postulate that the amplitude was an analytic function of ℓ for *all* ℓ was that the undetermined polynomial in the Mandelstam representation disappeared. Since it was the presence of this polynomial that undermined the thought that the holistic nature of the S-matrix precluded fundamental particles, the plausibility of the intuition was thereby restored. But this *intuition* was of course no more than that; barring a solution of the equations this disappearance of the polynomial, though lending hope to the anti-fundamentalists, was in itself inconclusive. The much more important implication of the postulate was that *all* particle singularities, including those corresponding to the lowest partial waves, should appear as Regge poles in the amplitude. Not only does this give a precise formal meaning to the idea that all strongly interacting particles are composite, and hence that there are no *fundamental* particles

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[65] "The key problem of bootstrap dynamics is to find a technique of continuing that Froissart-Gribov formula down to values of angular momentum for which poles appear. Not general technique has yet been developed." (Chew [1966], p60.)

[66] Some progress on this issue was made, but the argument was long and complicated and in any case only applied to elastic processes. See for example the discussion of Martin's proof in Collins and Squires [1968], p141.

[67] "Assuming analyticity in *s*, the Froissart limit evidently precludes such a special status for any physical [ℓ] larger than 1, but to date the general principles [so far introduced] have not been shown to ensure that these three lowest [ℓ] values must be ordinary citizens in a nuclear democracy. … It may eventually develop that complete democracy is the only way to achieve maximal analyticity of the first degree. Currently, however, it seems necessary to invoke an additional postulate." (Chew [1966], p54.)

[68] Ibid., p55.

amongst them, it also renders the anti-fundamentalist hypothesis directly empirically testable.

The reasons for this are the following. As we have noted, maximal analyticity implies that the angular momentum is a continuous function of the energy, and as the energy increases a pole at a given value of ℓ moves along on its trajectory $\alpha(E)$. Hence poles that contribute to one wave are functionally related to poles in others. This gives rise to distinctive asymptotic behaviour by Regge poles: it implies that it is the leading singularity in the crossed channel t that governs the asymptotic behaviour of the amplitude $A(s,t)$ as the direct channel energy s goes to infinity. On the other hand, poles that contribute to only one partial wave (and hence which are not Regge poles) give rise to delta-function type singularities and lack this behaviour. Being non–Regge, these poles can be taken as candidates for those corresponding to elementary particles.[69] Therefore it could be said that

> since bound states clearly lie on Regge trajectories whilst CDD [i.e. non-Regge] poles, in particular partial waves, give rise to Kronecker delta singularities in [the partial wave amplitudes], Regge theory offers a precise way of distinguishing between composite and elementary particles, and therefore of testing the idea of nuclear democracy that there are no elementary particles.[70]

Thus it is the postulate of maximal analyticity upon which Regge theory rests that results in the testability of nuclear democracy. And Chew did indeed stake his entire anti-fundamentalist capital on the hope that the Regge asymptotic prediction would prevail over the rival fundamentalist delta-function.

> Thus there exists at least one possible path for experimental demolition of the hadronic bootstrap: the discovery of non-Regge poles among hadrons.[71]

[69] For a more precise discussion of the relationship, see Jacob and Chew [1964a], pp.121-123.
[70] Squires [1971], p74.
[71] Chew [1968], p764

It was the asymptotic Regge behaviour that rendered maximal analyticity of the second kind, and hence nuclear democracy, falsifiable; and falsified it was. Despite much hope that it was down to experimental error, the pion consistently refused to exhibit Regge behaviour.[72] Since the anti-fundamentalist hypothesis was equated with the absence of deviations from Regge behaviour, we may say that nuclear democracy may from this point be regarded as *falsified*.[73]

Part 6 Conclusions

Having now examined the grounds upon which S-matrix theorists justified their claim that there are no fundamental hadrons, let us briefly retrace our steps. We first of all noted the contemporary philosophy is pervaded with the assumption that chains of dependence relations must terminate. We then marshalled the S-matrix theory in the service of demonstrating that one could conceivably find oneself in the position of denying this on naturalistic grounds. We then described a composite, or bound state, in S-matrix theory and showed that the compositional relations applicable there form partial orders. Three increasingly compelling arguments as to why this compositional ordering should be non-well-founded were then advanced. The *argument from superfluousness* maintained that elementary particles may be eschewed on the grounds that they are not demanded by the formalism. But of course, that there is no *need* for elementary particles does not preclude them. The *argument from holism* suggested that such particles are indeed positively forbidden. But in lieu of an exact solution to the equations there could be no demonstration of this, and indeed the need to deal with the

[72] Chew [1967], p189; see also Cushing [1990], p164.

[73] Of course, in reality things are rarely regarded as so simple. Cushing's interpretation of the demise of the 'autonomous S-matrix programme' is due to "degenerating Regge phenomenology" and exemplifies a "degenerating research programme in Lakatos' sense of that word" (Cushing [1990,] p154). Likewise, according to Redhead "the bootstrap programme was not so much *refuted* as *overtaken* by the new fundamentalist approach involving involving truly basic constituents like quarks and gluons" ([2005], p.573). While these are no doubt accurate assessements of the history, given Chew's insistence that non-Regge poles represented the 'demolition' of the theory and the argument underpinning it, I think we can hazard that these experiments may be taken to have a far more destructive significance for the claims of the programme than they perhaps in practice did. In any case, little of any consequence for our purposes hangs on this (doubtlessly naive) impression. (Note that other avenues of refutation were also envisaged as possible, such as the violation of the Levinson theorem regarding the high-energy phase shifts. See Collins and Squires ([1968], p145), and Jacob and Chew ([1964a], p127).)

divergence of the amplitude suggested against it. However, the *argument from analyticity* was able to place limits on this divergence and in so doing translate the S-matrix's flagship claim into a precise conditional: if the postulate of maximal analyticity of the second kind is true, then there are no fundamental hadrons. This postulate was motivated on a variety of independent grounds, and admitted of a precise empirical test. The failure of this test tells us that the picture S-matrix theory offers – of hadrons being composed of other hadrons without end – is after all not true of this world.[74]

As an example of an *internal* argument against fundamentality, the above study demonstrates a number of things.

1. *Fundamentality questions can be empirical questions*. We need *not* view questions of whether it is necessary or otherwise that "chains of dependence must terminate" as the exclusive purview of armchair speculation. Now, given that S-matrix theory is so closely connected with phenomenology – its central theoretical component is after all the S-matrix's compendium of observable results – I would hazard that such a high degree of direct *empirical* contact with fundamentality-related propositions is not something that we should expect to be a general feature of internal arguments against fundamentality. (There does not seem to be anything comparable in the effective field programme, for example.) Nevertheless, S-matrix theory does provide us with an example of the attempt to frame what many would take to be a quintessentially metaphysical hypothesis – the infinite divisibility of (a certain kind of) matter and hence the existence of gunk – in thoroughgoingly empirical terms.

2. *Arguments against fundamentality need not be meta-inductions*. The S-matrix argument against fundamentality proceeded entirely from within its own deductive system. Thus it has been demonstrated that it can be the internal logic of a physical theory – the implications of its system of postulates – that can be used to furnish a

[74] Of course, today we do know that all hadrons *are* composite (indeed now 'hadrons' are now usually defined as those particles that are composed of quarks); we don't know whether the quarks they are composed of have further structure or not. But this of course is compatible with the falsity of the S-matrix proposition that all hadrons are composites *of other hadrons.*

denial of the supposed necessity of fundamental entities, and it follows from this that arguments against fundamentality need not trade in speculative assumptions regarding the progress of future physics. (It may also be inferred from the fact that the anti-fundamentalist hypothesis became equated with a postulate concerning the extension of angular momentum functions to complex values that there may be nothing *a priori* obvious about the guise in which an anti-fundamentality hypotheses might appear in!) The above discussion also makes salient something implicit in the very concept of an internal argument against fundamentality, namely that the anti-fundamentalist must in all cases be committed to *something* at least *methodologically* fundamental: the set of physical *principles* from which their ontologically anti-fundamentalist conclusions follow. As mentioned, for example, Chew was very open about the fact that his key analyticity postulate in particular was simply taken as axiomatic, issuing from no more fundamental source. This defining aspect of the internal approach has another important consequence.

3. *Internal arguments against fundamentality are limited.* By definition, internal arguments proceed from a set of postulates, formulated by means of a finite set of predicates. As such, there is only a certain amount of qualitative variation permitted in the descending ontologies they are capable of fully describing. Those who were hoping that we could have naturalistic grounds for thinking that the world unfolds into stories as different as classical and quantum mechanics again and again *ad infinitum* as we descend more deeply into matter are likely to be disappointed.[75] The picture that S-matrix theory presents us with, for example, is one in which compositional chains go on forever, but the *types* of particles that feature in these chains recur *ad infinitum*.[76] Although it is not clear to me at this point how best to define the fundamentality of properties, it seems at least plausible that S-matrix theory implies fundamental *properties* even though no fundamental *particles*. Schaffer refers to worlds such as this – worlds in which the property structure repeats itself indefinitely as we plunge deeper down chains of mereological dependence – as 'boring'.[77] Although the degree of

[75] Such a world is conjectured and described by David Bohm ([1957], chapter 5).
[76] As Veneziano puts it, "It could be that we have an infinite variety of particles that interact with each other in a small region of space... in such a way as to form bound (or resonating) states that possess again the properties of the constituents" ([1969], p36. (Quoted in Gale [1974].)
[77] Schaffer [2003], p505.

homogeneity in the descending sequence need not be quite so dramatic as this in general – again, it doesn't seem to be the case in the effective field picture – the kind of anti-fundamentality that we can hope to establish on naturalistic grounds can only ever be a sort of 'half-way house' in which the *framework* stays the same even as the dependence structure, in some sense, never ends. Given that internal arguments against fundamental entities, or fundamental laws, are always relative to a set of physical principles which are at least *treated* as fundamental, the natural issue to address at this point is the status of fundamental *principles* in characterizing an ontologically fundamental basis and the nature of ontological dependence relations that might be said to exist between principles and particles or laws. But that is a matter for another occasion.

Turning back to present concerns, and in spite of the limitations and lacunae outlined above, what the argument just adduced demonstrates is that that the intuition that "chains of dependence must terminate" is not one that the naturalized metaphysician need share. While there remains a great deal to say on the relation between the *frameworks* that internal arguments must take as fundamental and the way in which fundamentality is understood in contemporary metaphysics generally, that, I believe, is a philosophical achievement in itself.

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